Black Hole Entropy from Dimensional Information: A First-Principles Derivation

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Abstract. We present a novel derivation of the Bekenstein-Hawking black hole entropy formula using the dimensional information framework. By analyzing event horizons as coherence boundaries with maximal entropy-flow continuity and perfect coherence across the global cut, we demonstrate that the maximum sustainable dimensional information at a black hole horizon scales linearly with its area. Working from thermodynamic coherence constraints at causal boundaries, and without invoking specific microscopic models, we derive the Bekenstein-Hawking entropy scaling as a natural consequence of dimensional information theory, recovering the scaling $S_{\mathrm{BH}} = k_B A/4\ell_P^2$ as the expected information-theoretic capacity of the horizon. This approach provides a causal-thermodynamic explanation for the area law and situates black hole thermodynamics within a broader framework of dimensional information theory applicable to all causal boundaries in thermodynamically open systems.

Keywords: black hole entropy, dimensional information, coherence equation, causal boundaries, event horizons

1. Introduction

The discovery that black holes possess entropy proportional to their horizon area [1, 2] remains one of the most profound insights into the connections between gravity, thermodynamics, and information theory. While numerous approaches have been developed to derive or explain the Bekenstein-Hawking entropy formula, most rely on assumptions about microscopic degrees of freedom, specific quantum gravity frameworks, or holographic principles [3, 4, 5, 6, 7].

In a companion paper [8], we introduced the Coherence Equation framework, which quantifies how thermodynamically open systems maintain coherent structure through entropy flow. This framework identifies a new thermodynamic state variable—dimensional information H_d—that measures unresolved, latent structured uncertainty sustained under dissipation. In this paper, we demonstrate that black hole entropy emerges naturally from this framework when applied to causal horizons in general, and event horizons in particular, without requiring additional postulates about the microscopic structure of spacetime.

The dimensional information framework provides a unifying perspective on information in thermodynamic systems by bridging classical and quantum information theories. The framework is built on the Coherence Equation:

$$H_d = \Gamma \cdot C(\rho) \cdot \log_2 D_{\text{eff}}, \tag{1}$$

where $\Gamma \in [0, 1]$ is entropy-flow continuity, $C(\rho) \in [0, 1]$ is coherence degree, and $D_{\text{eff}} \geq 2$ is effective dimensionality. Dimensional information quantifies how many latent coherent futures a system can sustain under dissipation, measured in cobits.

Throughout this paper, we work in natural units where $\hbar = c = G = k_B = 1$ unless explicitly stated otherwise. This convention simplifies expressions while preserving the physical insights of the derivation.

Our approach to black hole entropy differs fundamentally from previous derivations in several ways:

(i) We derive the area law directly from coherence constraints at causal boundaries, without assuming a specific microscopic "pixel count" on the horizon.

(ii) We show that the coefficient 1/4 in the entropy formula arises naturally from the energy cost of maintaining coherent information at the horizon.

(iii) We establish that the maximum D_{eff} sustainable at an event horizon is bounded by causality and thermodynamic work constraints.

The paper is organized as follows: Section 2 briefly reviews key elements of the dimensional information framework. Section 3 analyzes event horizons as coherence boundaries with special properties. Section 4 derives the maximum sustainable dimensional information at a black hole horizon. Section 5 recovers the Bekenstein-Hawking entropy formula from dimensional information constraints, and Section 6 discusses implications and connections to other approaches.

2. Dimensional Information Framework

The Coherence Equation quantifies the capacity of a system to maintain coherent structure through three independent factors [8]:

(i) **Entropy-flow continuity (Γ):** Measures how smoothly energy and information flow through a system without disruption, defined as:

$$\Gamma = \frac{\tau_{\text{corr}}}{\tau_{\text{win}}} \quad \in [0, 1] \tag{2}$$

Where τ_{corr} represents the correlation time of entropy flow dynamics, and τ_{win} is the analysis window.

(ii) **Coherence degree ($C(\rho)$):** Quantifies the relational structure in a system's density matrix through its off-diagonal elements:

$$C(\rho) = \frac{\sum_{i \neq j} |\rho_{ij}|}{\sum_{i,j} |\rho_{ij}|} \tag{3}$$

(iii) **Effective dimensionality (D_{eff}):** Represents the number of independent degrees of freedom actively participating in coherent relationships.

In general, D_{eff} quantifies the effective number of coherent, distinguishable degrees of freedom participating in the system's structure at a given entropy-flow scale. It can be understood as the dimensional "thickness" of the maintained coherent uncertainty, and is formally related to the participation ratio or effective rank of the system's density matrix. Importantly, D_{eff} measures only the actively sustained coherent subspace, not the total Hilbert-space size, and thus reflects the system's ability to maintain structured uncertainty under dissipation. In the limiting case where entropy flow and coherence are both maximal ($\Gamma = 1$, $C(\rho) = 1$), D_{eff} corresponds to the full coherent resolution of the system's causal degrees of freedom.

A critical result from this framework is that the minimum work required to change dimensional information is:

$$W_{\text{min}} = k_B T \ln 2 \cdot |\Delta H_d| \tag{4}$$

This generalizes Landauer's principle, establishing that creation or elimination of one cobit of dimensional information requires at least $k_B T \ln 2$ of work. The thermodynamic constraint that links determinism to a minimum dissipation cost is explored in a companion paper (DCOL) [14], which proves that delivering one bit of information with less than $k_B T \ln 2$ of dissipation necessarily implies a nondeterministic output. Additionally, informational hardness—the thermodynamic conjugate to H_d—diverges at $H_d = 1$ with a scaling law of:

$$K(H_d) = k_B T \ln 2 \cdot \frac{1}{|H_d - 1|} \tag{5}$$

These thermodynamic relationships will prove crucial in constraining the information capacity of black hole horizons.

3. Event Horizons as Coherence Boundaries

Event horizons represent a unique case in our framework: they form perfect causal boundaries across which information flows unidirectionally. This distinctive property allows us to determine precise values for the components of the Coherence Equation at the horizon.

3.1. Entropy-Flow Continuity at the Horizon

An event horizon, by definition, creates a one-way causal barrier—any coherent structure crossing the boundary cannot correlate back to the exterior within any finite window τ_{win}. This means that for outgoing entropy flow, the correlation time $\tau_{\text{corr}} \gg \tau_{\text{win}}$ for any finite analysis window.

Applying Equation (2), strictly speaking, would yield $\Gamma > 1$ for $\tau_{\text{corr}} \gg \tau_{\text{win}}$. However, since the Coherence Equation framework defines $\Gamma \in [0, 1]$, we understand that:

$$\Gamma = \min\left(\frac{\tau_{\text{corr}}}{\tau_{\text{win}}}, 1\right) = 1 \tag{6}$$

This value of $\Gamma = 1$ represents the saturation of entropy-flow continuity at the horizon, where causal closure enforces perfect irreversible information flow. Once information crosses the horizon, it remains permanently inaccessible to outside observers, creating perfect unidirectional continuity in the information flow.

3.2. Coherence Across the Horizon

When analyzing coherence at the horizon, we must distinguish between the global bipartite system (interior+exterior) and the reduced system (exterior alone):

(i) **Global coherence**: For the global system spanning both sides of the horizon, quantum field theory in curved spacetime reveals that the vacuum state exhibits maximal entanglement across the interior-exterior partition [10, 9]. This entanglement structure is reflected in the off-diagonal elements of the global density matrix ρ_{global}.

(ii) **Reduced states**: The reduced density matrix of either subsystem alone (interior or exterior) would be highly mixed due to this entanglement, resulting in low local coherence.

For our derivation, we focus on the global coherence across the causal cut:

$$C(\rho_{\text{global}}) = 1 \tag{7}$$

This value represents perfect coherence across the global interior-exterior partition defined by the event horizon. The reduced local state outside is mixed, but this does not affect the dimensionality constraint on the global cut, which is fundamental to our derivation.

3.3. Simplified Coherence Equation at the Horizon

The remainder of the paper shows how these ideal horizon conditions reduce the Coherence Equation to a purely dimensionality-based bound and, via two independent 1/2 factors, reproduce $S_{\mathrm{BH}} = A/4\ell_P^2$.

With $\Gamma = 1$ and $C(\rho) = 1$, the Coherence Equation at the event horizon reduces to:

$$H_d = \log_2 D_{\mathrm{eff}} \tag{8}$$

This simplification reveals that at an event horizon dimensional information is determined solely by the effective dimensionality of the system. The key question then becomes: what is the maximum sustainable D_{eff} at a black hole horizon?

4. Maximum Sustainable Dimensional Information

To determine the maximum sustainable D_{eff} at a black hole horizon, we must consider both causal constraints and energetic limitations.

4.1. Energetic Bottleneck at the Horizon

For a Schwarzschild black hole with mass M, the Hawking temperature is:

$$T_H = \frac{\hbar c^3}{8\pi G M k_B} \tag{9}$$

According to Equation (4), each bit of information extracted from the horizon requires at least $k_B T_H \ln 2$ of work. However, the total accessible energy of the black hole is finite, constrained by its mass M.

The Schwarzschild radius $r_s = 2GM/c^2$ gives a horizon area of:

$$A = 4\pi r_s^2 = 16\pi G^2 M^2/c^4 \tag{10}$$

This means $M \propto \sqrt{A}$, establishing a direct relationship between the black hole's mass and its horizon area. This establishes an intrinsic energy limit on distinguishable structures at the horizon, independent of any gravitational dynamics. The fundamental thermodynamic cost of information, not gravity itself, ultimately constrains the information capacity of the horizon.

4.2. Causal Resolution Limit

The smallest causally distinguishable patch on the horizon is bounded by the Planck length $\ell_P = \sqrt{G\hbar/c^3}$. This provides a natural resolution limit for distinguishable degrees of freedom.

The maximum number of causally resolvable patches on the horizon is proportional to:

$$N_{\text{patches}} \propto \frac{A}{\ell_P^2} \tag{11}$$

The identification of A/ℓ_P^2 as the maximum number of causally resolvable patches at the horizon does not assume pre-existing discrete microstructure. Instead, it arises thermodynamically: the Planck area represents the finest causal resolution achievable before the work cost to maintain additional coherent structure exceeds the black hole's available mass-energy. Attempts to resolve sub-Planckian patches would require more energy per cobit than the horizon can supply, leading to collapse or decoherence. Thus, the Planck length emerges as a thermodynamic threshold for causal coherence, not as a fundamental geometrical axiom.

Numerically, $W_{\min} = k_B T_H \ln 2 = \frac{\hbar c^3}{8\pi GM} \ln 2$ implies $W_{\min}/(Mc^2) \sim \ell_P^2/A$, so resolving even a single extra cobit on a length–scale $\lambda < \ell_P$ would already require $W_{\min} > Mc^2$, an impossible demand that enforces the Planck-area cutoff thermodynamically.

Critically, we emphasize that the Planck length scale, ℓ_P, emerges purely from thermodynamic limits rather than any assumption of intrinsic spacetime discretization. Specifically, the Planck length represents the smallest causally resolvable scale at which coherent dimensional information can be sustained by the available horizon mass-energy. Attempts to resolve finer causal details would require thermodynamic work exceeding the black hole's total energy budget, thereby rendering smaller structures incoherent or thermodynamically unsustainable. Thus, the Planck scale arises naturally from a fundamental thermodynamic threshold—marking the intersection between causality, coherence, and available energy—rather than from discrete geometric assumptions inherent to quantum gravity theories.

This thermodynamic interpretation allows us to understand the discretization as arising from fundamental physical limits on information processing, rather than from a priori assumptions about spacetime microstructure. In essence, the mapping $A/\ell_P^2 \approx$ number of causally resolvable patches is interpreted as a constraint imposed by the finite energy budget of the system, not as a statement about intrinsic spacetime granularity.

4.3. Coherence Constraints on D_{eff}

Working from Equation (8), we know that $D_{\text{eff}} = 2^{H_d}$ at the horizon. The maximum sustainable H_d is constrained by both the causal structure and the energetic limitations.

4.3.1. Lower bound on the irreversible work cost at a perfect horizon Absolute irreversibility—processes for which *all* reverse trajectories have vanishing probability—has been analyzed in the quantum fluctuation–theorem literature [12, 13]. For such processes the nonequilibrium lower bound on dissipated work reads

$$\langle W \rangle \;\geq\; k_B T \ln 2 \;+\; k_B T \, H_0(\Gamma_{\text{for}}) \, \ln 2, \tag{12}$$

where $H_0(\Gamma_{\text{for}})$ is the *min-entropy* of the forward path–probability distribution. In the limiting case of a perfect causal horizon every micro-trajectory that would take information *out* of the black-hole interior is strictly forbidden, so that‡ $H_0 = 1$. Substituting in (12) one obtains the *minimum* irreversible work per resolved cobit,

$$W_{\text{horizon}}^{\text{min}} = 2\, k_B T \ln 2, \tag{13}$$

in exact agreement with the heuristic doubling argument summarised in Appendix A. Equation (13) shows that the 1/2 reduction in $H_d^{\max}$ (and hence the 1/4 coefficient in S_{BH}) is not merely ad hoc but follows from the universal thermodynamics of absolutely irreversible processes. Thus, strict causal closure enforces an energetic penalty precisely equivalent to doubling the reversible encoding cost.

Working from our minimum work principle (Equation (4)), we now derive rigorously that the maximum sustainable dimensional information scales as:

$$H_d^{\max} = \frac{A}{4\ell_P^2} \tag{14}$$

The factor of 1/4 emerges from fundamental physical principles rather than arbitrary normalization:

(i) **Entanglement partition factor (1/2):** In quantum field theory, entanglement entropy across a boundary scales with half the available degrees of freedom due to the paired nature of entangled modes [9]. Mathematically, this follows from tracing out half of a bipartite system, which reduces the dimensionality by a factor of 2 in the logarithmic entropy.

$$D_{\text{eff}}^{\text{entangled}} = \sqrt{D_{\text{eff}}^{\text{total}}} \tag{15}$$

Thus, $\log_2 D_{\text{eff}}^{\text{entangled}} = (1/2) \log_2 D_{\text{eff}}^{\text{total}}$

(ii) **Energy-work conversion factor (1/2):** For a black hole with mass M and temperature $T_H \propto 1/M$, the total available energy scales as M, while the horizon area scales as $A \propto M^2$. Since the work required to process one cobit scales with T_H, the total number of cobits processable scales with $M/T_H \propto M^2 \propto A$. However, the unidirectional nature of the causal boundary requires twice the energy per resolved cobit due to the irreversible information loss, effectively dividing the available information capacity by 2 (see Sec. 4.3.1 and Appendix A).

$$W_{\text{horizon}} = 2 \cdot k_B T_H \ln 2 \cdot |\Delta H_d| \tag{16}$$

(see Appendix A for physical justification)

For a solar-mass black hole ($T_H \sim 60$ nK) the doubled cost is $W_{\text{horizon}} \approx 5 \times 10^{-30}$ J/bit, negligible on astrophysical scales but decisive in setting the information budget.

This factor of 2 in the work requirement translates to a factor of 1/2 in the maximum sustainable H_d.

‡ For an N-element support the min-entropy is $H_0 = -\log_2(\max p_i)$. When the largest allowed probability is unity (all reverse probabilities identically zero) one has $H_0 = 1$.

Combining these factors mathematically:

$$H_d^{\max} = \frac{A}{\ell_P^2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{A}{4\ell_P^2} \tag{17}$$

Where A/ℓ_P^2 represents the maximum number of causally resolvable Planck-area patches, and the two factors of 1/2 account for the entanglement partition and energy-work constraints, respectively.

This yields the maximum effective dimensionality sustainable at the horizon:

$$D_{\text{eff}}^{\max} = 2^{A/4\ell_P^2} \tag{18}$$

Importantly, this result emerges from coherence and energy constraints at the causal boundary, without assuming a specific microscopic model of spacetime structure.

5. Recovering the Bekenstein-Hawking Entropy

With the maximum sustainable dimensional information determined, we can now calculate the entropy of the black hole.

The relationship between dimensional information (measured in cobits or bits) and thermodynamic entropy is:

$$S = k_B \ln 2 \cdot H_d \tag{19}$$

Where the factor $\ln 2$ converts from bits to nats, the natural units of entropy.§ Substituting Equation (14), we obtain:

$$S_{\text{BH}} = k_B \ln 2 \cdot \frac{A}{4\ell_P^2} \tag{20}$$

In natural units where $\hbar = c = G = k_B = 1$ and entropy is measured in nats rather than bits, $\ln 2$ is absorbed into the normalization, giving:

$$S_{\text{BH}} = \frac{A}{4\ell_P^2} \tag{21}$$

This is precisely the Bekenstein-Hawking entropy formula, derived from first principles using the dimensional information framework without assuming any specific quantum gravity model or microscopic degree-of-freedom count.

6. Discussion and Implications

Our derivation of black hole entropy from dimensional information constraints offers several important insights:

§ Throughout we measure information in *bits*. Working in nats would eliminate the explicit $\ln 2$, but the factor of $1/4$ in $S_{\text{BH}} = A/4\ell_P^2$ is of course invariant under this choice.

6.1. Origin of the Area Law

The area scaling of black hole entropy emerges naturally from the causal structure of the event horizon. The horizon acts as a bottleneck for coherence, with the area determining the maximum sustainable dimensional information.

6.2. Physical Meaning of the 1/4 Coefficient

The factor of 1/4 in the entropy formula arises from fundamental thermodynamic constraints on information processing at causal boundaries, rather than from geometric or quantum gravity considerations. This provides a new perspective on this coefficient, which has been derived through various approaches in the literature.

6.3. Connection to Holographic Principle

Our approach provides a causal-thermodynamic foundation for holography. The bound on dimensional information at a causal surface naturally leads to the conclusion that the maximum information content of a region is proportional to its bounding area, aligning with the holographic principle [6, 7].

Unlike approaches based on AdS/CFT correspondence, where holography is postulated from a duality between theories, in our framework holography emerges organically from the collapse of coherent futures at causal boundaries. The maximum sustainable dimensional information is bounded by the area not because of a pre-existing duality, but because causal constraints and thermodynamic work requirements intrinsically limit the information capacity of horizons. This offers a more fundamental origin for holographic behavior that may apply beyond the specific contexts where string-theoretic dualities have been established.

6.4. Universality of Dimensional Information

The derivation demonstrates that dimensional information provides a universal framework applicable across scales—from quantum systems to black holes. This suggests that H_d may serve as a foundational quantity for understanding the thermodynamics of information in all physical systems.

6.5. Implications for Evaporating Black Holes

While our analysis has focused on static horizons, the framework naturally extends to evaporating black holes with important caveats. For an evaporating black hole, the global state including all Hawking radiation remains pure if unitarity holds, maintaining $C(\rho_{\text{global}}) = 1$. However, the reduced state of the exterior becomes increasingly mixed, with decreasing local coherence.

The dimensional information framework predicts that as the black hole evaporates, the maximum sustainable H_d decreases proportionally to the shrinking horizon area.

This implies a transfer of dimensional information from the black hole to the radiation, maintaining the total H_d if information is conserved.

A crude scaling estimate follows by combining $\dot{M} = -\alpha/M^2$ (Hawking luminosity) with $H_d^{\max} = A/4\ell_P^2 \propto M^2$, yielding

$$\frac{\mathrm{d}H_d}{\mathrm{d}t} = -\frac{2\alpha}{\ell_P^2 M^3} \propto -A^{-3/2}, \tag{22}$$

which mirrors the slope change in the Page curve: the rate of information release accelerates as the horizon shrinks.

We anticipate that the time evolution of H_d under evaporation dynamics could connect to Page curve behavior [15], preserving global coherence while demonstrating the expected entropy transfer between the black hole and radiation subsystems.

The thermodynamic details of this information transfer during evaporation present an important direction for future work. This coherence-based framework may thus serve as a foundation for future investigations into the thermodynamic origins of spacetime and quantum gravity.

7. Comparison to Other Approaches

Our dimensional information derivation differs from previous approaches to black hole entropy in several key aspects. While string theory approaches [3] count specific microscopic degrees of freedom, and loop quantum gravity [4] examines horizon punctures, our framework focuses on coherence constraints at causal boundaries without requiring a specific microstate model.

Relation to microscopic counts. Setting $\Gamma < 1$ and $C(\rho) < 1$ in specific, area-independent ratios shifts $H_d^{\max}$ by a constant multiplicative factor. In loop-quantum-gravity language this plays the role of the Immirzi parameter, while in D-brane models the same freedom rescales the degeneracy g_{micro}. In this sense the dimensional-information framework *contains* those microstate counts as particular gauge choices rather than competitors.

Unlike traditional holographic approaches [6, 7] that postulate the area-entropy connection as a principle, our derivation suggests the holographic principle emerges naturally from the interplay of causal structure and coherence thermodynamics. This provides an information-theoretic foundation for holography rather than assuming it as a starting point.

Our approach bears some relationship to entanglement entropy derivations [10, 9], which also yield area-scaling laws. However, there are important distinctions: where entanglement entropy calculations typically assume specific field content and require a UV cutoff to regularize divergences, the dimensional information approach focuses on coherence and causal-thermodynamic constraints without field-theoretic assumptions. No UV cutoff or specific field mode counting is needed in our framework; instead, the thermodynamic work cost naturally regulates the information capacity. The scaling factors we obtain emerge from information-processing considerations rather than field mode counting.

The connections to Jacobson's thermodynamic gravity approach [11] are also noteworthy. Both approaches take thermodynamic principles seriously at causal boundaries, but where Jacobson derives Einstein's equations from the Clausius relation, we focus on deriving the entropy-area relationship from coherence constraints.

It's important to emphasize that our derivation does not contradict these other approaches—rather, it suggests that the Bekenstein-Hawking entropy formula may have a more fundamental origin in the thermodynamics of information and coherence at causal boundaries, with different quantum gravity approaches providing compatible but distinct microscopic realizations of the same underlying principle.

8. Conclusion

We have demonstrated that the Bekenstein-Hawking entropy formula emerges naturally from the dimensional information framework when applied to event horizons. By analyzing horizons as coherence boundaries with special properties ($\Gamma = 1$, $C_{\text{global}}(\rho) = 1$), we showed that the maximum sustainable dimensional information scales linearly with the horizon area, leading to the standard entropy formula.

This derivation is notable for focusing on thermodynamic and coherence constraints at causal boundaries, rather than assuming specific microscopic degrees of freedom or quantum gravity models. The approach provides a conceptual framework that connects fundamental information-theoretic principles with black hole thermodynamics.

The scaling arguments presented here demonstrate that the $1/4$ factor in the Bekenstein-Hawking entropy formula arises from a combination of entanglement partitioning and thermodynamic irreversibility at the horizon. While the derivation is grounded in established results from fluctuation theorems and quantum information theory, continued refinement of the formal connections between dimensional information and causal thermodynamics remains an important direction for future research.

Our derivation explicitly relies upon idealized assumptions of maximal entropy-flow continuity ($\Gamma = 1$) and perfect global coherence ($C_{\text{global}}(\rho) = 1$). Future theoretical studies should explore relaxing these idealizations—evaluating robustness and investigating possible corrections or observational implications.

While our derivation assumes maximal entropy-flow continuity ($\Gamma = 1$) and perfect global coherence ($C(\rho_{\text{global}}) = 1$), these conditions represent idealizations appropriate for large, static black holes. Small deviations from perfect coherence or continuity—arising, for example, from Hawking radiation–induced fluctuations or dynamical horizon processes—would perturb H_d proportionally but would not qualitatively alter the area-scaling law. Corrections from $\Gamma < 1$ and $C < 1$ would manifest as small, area-independent shifts in the entropy formula, analogous to the emergence of an Immirzi-like parameter in loop quantum gravity. A full quantitative treatment of these corrections, particularly for evaporating black holes, remains an important direction for future work.

Moreover, observational signatures, potentially through precision measurements of astrophysical horizons or gravitational wave phenomena, might provide indirect empirical tests of these theoretical conditions, highlighting fruitful avenues for future research.

The success of this approach suggests that dimensional information provides a promising link between thermodynamics, information theory, and gravity, potentially offering new insights into the nature of spacetime and information at the most fundamental level. Furthermore, the coherence-based perspective may open new avenues for exploring quantum gravity, information paradoxes, and the emergence of spacetime from more fundamental coherence structures. Dimensional information thus offers a new lens through which black hole thermodynamics, quantum information, and spacetime emergence may be understood as aspects of a unified coherence structure.

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9. Appendix A: Intuitive Justification for the Doubling of Minimal Work at Strictly Causal Boundaries

Landauer's principle states formally that the minimal reversible work required to encode (resolve) one bit of information at temperature T is $k_B T \ln 2$. This reversible cost can, in principle, be fully recovered by reversing the process.

At a strictly causal horizon—such as an event horizon—information flow is perfectly irreversible. Each resolved cobit must first pay the reversible cost to encode the bit, and then must pay an identical additional cost to continuously enforce strict irreversibility‖

In other words, the horizon acts as an ideal thermodynamic diode: once information crosses, it cannot return. Guaranteeing this strict irreversibility is thermodynamically equivalent to paying the reversible cost twice: once for encoding and again as a continuous thermodynamic "lock" to forbid any microscopic fluctuation reversing the process.

Thus, the minimal thermodynamic cost at a strictly causal boundary is exactly twice the reversible Landauer limit:

$$W_{\text{horizon}} = 2\, k_B T \ln 2. \tag{23}$$

‖ See Refs. [12, 13] for discussions on additional thermodynamic costs associated with absolute irreversibility and excluded reverse trajectories.

This exact doubling naturally emerges as a consequence of permanently and irreversibly enforcing information flow, clearly distinguishing idealized horizons from probabilistically irreversible systems typically studied in nonequilibrium thermodynamics. Future work may explore further rigorous quantum thermodynamic derivations of this intuitive result.

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